CONSTRUCTION OF THE CREEP EQUATIONS OF MATERIALS WITH ALLOWANCE FOR THE SOFTENING STAGE IN UNIAXIAL DEFORMATION

V. N. Maklakov UDC 539.376

This article examines problems relating to the construction of equations describing the creep and rupture strength of metallic materials. Attainment of the limiting value of internal energy density is taken as the criterion of failure of an elementary volume of the given material. It is shown that the proposed model leads to a linear damage summation law with weight factors dependent on the loading history. The model makes it possible to describe the nonmonotonic character (with one or two local extrema) of the stress dependence of creep strain at the moment of failure. The validity of the model is demonstrated for several materials.

1. We will examine the unidimensional isothermal creep of metallic materials and establish several laws on the basis of tests conducted to failure. in the absence of a strain-hardening stage, the creep curve (constant stress σ) consists of two sections: a stage of steady-state flow (the second stage of creep), where the strain rate is constant and equal to the minimum value $\dot{p}_{min}$; a softening stage (the third stage of creep), where the rate increases from the minimum to its limiting value at the moment of failure [1]. We distinguish between the following cases of deformation.

1. The duration of the second stage is only a small part of the total time to rupture, and the creep rate $\dot{p}_*$ increases without limit at the moment of failure t_* [1, 2].

2. The duration of the second stage is only a small part of the time to rupture, and $\dot{p}_*$ is a finite quantity [3-5].

3. The durations of the second and third stages are comparable and $\dot{p}_*$ increases without limit [6, 7].

4. The durations of the second and third stages are comparable and $\dot{p}_*$ is a finite quantity [8, 9].

The system of kinetic equations describing the creep process [1]

$$\dot{p} = C\sigma^n (1 - \omega)^{-r}, \quad \dot{\omega} = B\sigma^s (1 - \omega)^{-q}, \tag{1.1}$$

where ω is the damage parameter. Here, $\omega = 0$ for the initial state and $\omega_* = 1$ at the moment of failure. Equations (1.1) describe case 1, since at $t > 0$ we have $\dot{p} > \dot{p}_{min} = C\sigma^n$ and $\dot{p}_* \to \infty$ as $\omega_* \to 1$.

The models in [3-5] can be used for case 2. The systems of equations proposed in [1, 2, 6] describe case 3. The results presented in these studies indicate that one of the models simultaneously describe all of the above cases of creep. The development of such a model is the goal of our investigation.

We will treat the structural parameter ω as a quantity which is connected with the creep rate and is unrelated to the strength properties of the material. This makes it unnecessary to use the condition $\omega_* = 1$. We represent the effective stresses as $\sigma/(1 - \omega)$ [1, 2] and we take the differential equation for creep rate in the form [2] $\dot{p} = C[\sigma/(1 - \omega)]^n$.

System (1.1) predicts an increase in the parameter ω for any $t > 0$, since $\dot{\omega} = F\sigma^m\dot{p}^\beta$, where $F = BC^{-q/r}$, $m = s - nq/r$, $\beta = q/r$. Meanwhile, an adequate description of the second stage requires satisfaction of the conditio $\omega(t) \simeq 0$, by virtue of the fact that the steady-state creep rate is a constant value. We thus take $\dot{\omega} = F\sigma^m(\dot{p} - \dot{p}_{min})^\beta$. This equation is in accord with the empirical fact that no structural changes are seen in the material during the second stage [10].

Let us examine the criterion of failure. The authors of [11, 12] presented results of fatigue tests conducted under isothermal conditions. The work done by the macroscopic external forces A and thermal energy dissipation Q were monitored during the deformation process. The following was established from these experiments. First of all, the graphs of Q(t) and A(t) turn out to be similar and have the similarity coefficient $k(\sigma_a)$: $Q(t)/A(t) = k(\sigma_a)$, where t is time and σ_a is the amplitude of

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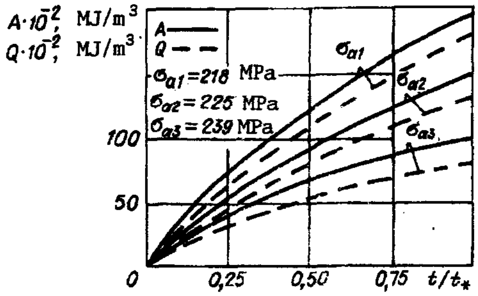

Fig. 1

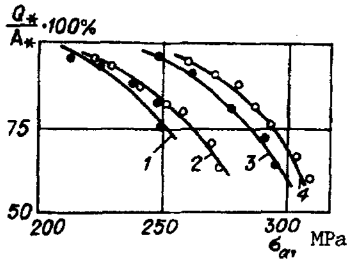

Fig. 2

the stress (Fig. 1, steel 45 [11]). Secondly, the ratio of Q_* to A_* at the moment of failure is a monotonic function $Q_*/A_* = f(\sigma_a)$ (Fig. 2, lines 1-4 for steels 25, 45, 2Kh13, 40Kh [11]). Thirdly, the critical value $U_* = A_* - Q_*$ at the moment of failure is independent of the loading history and is a constant of the material.

The energy balance equation (the first law of thermodynamics) [11, 12] shows that one part of A(t) is expended on a change in internal energy during deformation (i.e., energy is stored by the material), while the remaining part is dissipated into the surrounding medium in the form of heat. Thus, it is natural to choose internal energy density as the measure of damage accumulation

$$U = A - Q \tag{1.2}$$

(the minus sign indicates heat outflow; U, A, and Q are quantities referred to an elementary volume). We then connected the condition for failure of an elementary volume with the critical value U_*, which is a material constant [11, 12]. It is difficult to perform fatigue tests under high-temperature creep conditions, since the creep process is quasistatic and the specimen is placed in a test chamber with a temperature on the order of 400-500°C. This makes it almost impossible to use calorimetry to measure the heat that is released. We therefore decided to study creep by making use of certain findings from fatigue tests [11, 12] and then checking this data with our empirical data.

Since the similarity condition for Q(t), A(t) is satisfied at any moment of time – including at the moment of fracture – we have $k(\sigma) = f(\sigma)$. We thus obtain the following relation from (1.2)

$$U = [1 - f(\sigma)]A, \tag{1.3}$$

Here, the function $1 - f(\sigma)$ determines that part of the work A done on increasing damage content. It should be noted that, in an adiabatic process, there is no heat transfer between the specimen and the surrounding medium and $f(\sigma) = 0$. Assuming that the similarity condition for Q(t), A(t) is satisfied with a stepped change in load, we can use (1.3) to obtain an equation describing damage accumulation

$$\dot U = [1 - f(\sigma)]\dot A, \tag{1.4}$$

integrating this equation, we find the failure criterion

$$U_* = \int_0^{t_*} [1 - f(\sigma)]\,\dot A\,dt. \tag{1.5}$$

There is often no creep limit in high-temperature creep tests (the material is not deformed in the absence of an external load), so we approximate the unknown function by means of a power relation $f(\sigma) = 1 - a\sigma^{\varphi-1}$. Ignoring the elastic strains, we reason that $\dot A = \sigma \dot p$ [1]. Then, in accordance with the data reported in [4, 5], we introduce effective stresses into (1.4-1.5). Thus

$$\dot W = [\sigma/(1-\omega)]^{\varphi}\dot p, \quad W_* = \int_0^{t_*} [\sigma/(1-\omega)]^{\varphi}\dot p\,dt,$$

where $\dot W = \dot U/a$; $W_* = U_*/a$. It should be noted that at $\varphi = 1$ (adiabatic process) the given fracture criterion coincides with the well-known energy criterion in [13].

Finally, we write the system of governing equations in the form

$$\dot{p} = C\,[\sigma/(1-\omega)]^n, \quad p\,(0) = 0,$$
$$\dot{\omega} = F\sigma^m\,(\dot{p} - \dot{p}_{\min})^{\beta}, \quad \omega\,(0) = 0, \qquad \dot{W} = [\sigma/(1-\omega)]^{\varphi}\dot{p}, \quad W_* = \text{const}. \tag{1.6}$$

The proposed model leads to a constant creep rate at the steady-state stage of flow, due to the chosen structure of the equation for the parameter ω. System (1.6) makes it possible to describe both the case when $\dot{p}_* \to \infty$ (as $\omega_* \to 1$) and the case when $\dot{p}_*$ is a finite quantity (when $\omega_* < 1$).

At $t > 0$, the function $\omega(t) = 0$ is the solution of the second equation of system (1.6). The softening stage is not described in this case. We find the derivative ψ_ω' from the right side of the second equation of this system: $\psi_\omega' = FC^\beta\sigma^{m+n\beta}n\beta\,[1-(1-\omega)^n]^{\beta-1}/(1-\omega)^{n\beta+1}$. It is evident from this that the initial equation can have a nontrivial solution when $\beta < 1$ and $\omega(0) = 0$, since the condition of the uniqueness theorem [14] is violated.

Integration of system (1.6) leads to integrals of a differential binomial, so that the solution is expressed in finite form only in certain special cases [15]. For example, this is possible only when $n = -1/(\beta - 1)$, since for creep $n \geq 3$ [1]. Let us examine the effect of two-stage loading on the material: the stress σ_1 acts over the time interval from 0 to t_1, while the stress σ_2 acts from t_1 to the moment of failure t_*. Integrating system (1.6) with the satisfaction of this condition for material constants n and β, we arrive at a linear damage-summation law with weight factors dependent on the loading history:

$$t_1\,[1-(1-\omega_{*1})^n]^{1/n}/t_{*1} + t_2\,[1-(1-\omega_{*2})^n]^{1/n}/t_{*2} = [1-(1-\omega_{*12})^n]^{1/n}.$$

Here, ω_{*1} and ω_{*2} are the structure parameters at the moment of failure with constant σ_1 and σ_2, respectively; ω_{*12} is the structure parameter at the moment of failure for the specified two-stage loading; $t_2 = t_* - t_1$ is the time of action of the stress σ_2. It should be noted that the models in [1-3, 5, 6] do not have this feature, since it was shown in [16] that if an equation with separable variables is used for ω and if $\omega_* =$ const, then the model leads to the standard linear damage-summation law [1].

In the case when the duration of the stage corresponding to steady-state flow is unimportant, we can drop the requirement that the condition $\omega(t) \simeq 0$ be satisfied at this stage of deformation. We then obtain the simplified system

$$\dot{p} = C\,[\sigma/(1-\omega)]^n, \quad p\,(0) = 0,$$
$$\dot{\omega} = F\sigma^m\dot{p}^{\beta}, \quad \omega\,(0) = 0, \qquad \dot{W} = [\sigma/(1-\omega)]^{\varphi}\dot{p}, \quad W_* = \text{const}, \tag{1.7}$$

which can be integrated analytically with constant σ:

$$p_* = \frac{C^{1-\beta}\sigma^{n-n\beta-m}}{F\,(n\beta - n + 1)}\left\{1 - \left[1 - \frac{W_*F\,(n\beta - n - \varphi + 1)}{C^{1-\beta}\sigma^{\varphi+n-m-n\beta}}\right]^{(n\beta-n+1)/(n\beta-n-\varphi+1)}\right\}; \tag{1.8}$$

$$t_* = \frac{C^{-\beta}\sigma^{-n\beta-m}}{F\,(n\beta + 1)}\left\{1 - \left[1 - \frac{W_*F\,(n\beta - n - \varphi + 1)}{C^{1-\beta}\sigma^{\varphi+n-m-n\beta}}\right]^{(n\beta+1)/(n\beta-n-\varphi+1)}\right\},$$

$$\omega_* = 1 - \left[1 - \frac{W_*F\,(n\beta - n - \varphi + 1)}{C^{1-\beta}\sigma^{\varphi+n-m-n\beta}}\right]^{1/(n\beta-n-\varphi+1)}. \tag{1.9}$$

Here, $\omega_* = \omega_*(\sigma)$ is the structure parameter at the moment of failure t_*.

Equation (1.9) has two asymptotes in logarithmic coordinates. One asymptote corresponds to brittle fracture $\ln\sigma = \{-\ln t_* - \ln\,[FC^\beta\,(n\beta + 1)]\}/(n\beta + m)$, while the other corresponds to ductile fracture $\ln\sigma = [-\ln t_* + \ln\,(W_*/C)]/(\varphi + n)$. The point of intersection of the asymptotes $\sigma_k = [W_*FC^{\beta-1}(n\beta + 1)]^{1/(\varphi+n-m-n\beta)}$ determines the boundary between brittle and ductile fracture.

The finiteness of p_* and ω_* and the condition of upward convexity of the rupture-strength curve lead to the following restrictions on the material constants of model (1.7): $-n\beta - m \geq 0$, $n - n\beta - m \geq 0$, $\varphi + n - n\beta - m > 0$, $\varphi + n - n\beta - 1 > 0$. It follows from these inequalities that the parameter φ can be either negative or positive.

Relation (1.8) has an increasing section in the case of low stresses (brittle fracture), while $p_* \sim \sigma^{n-n\beta-m}$. We will examine the case of high stresses. We differentiate (1.8) with respect to σ and expand the result into a series. Limiting ourselves

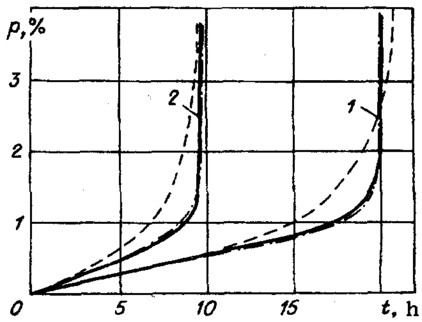

Fig. 3

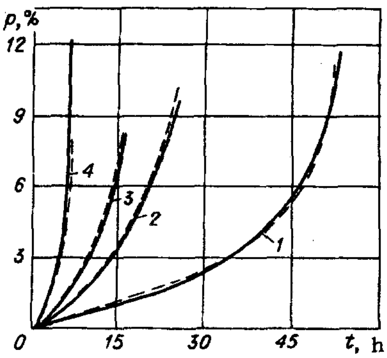

Fig. 4

to the first two terms of the series, we equate the expression to zero. We then find the stress $\sigma_0 = [W_* FC^{\beta-1}/(\varphi + n - m - n\beta)]^{1/(\varphi+n-m-n\beta)}$, at which the derivative $(p_*)_\sigma'$ changes sign. This stress is a local maximum at $\varphi > 0$ and a local minimum at $\varphi < 0$. Thus, the proposed model makes it possible to describe the nonmonotonic character of the relation $p_* = p_*(\sigma)$ with one or two local extrema.

2. Estimates of the material constants of systems (1.6-1.7) are made on the basis of the condition of the minimum of the functional

$$J = \frac{1}{k-2} \sum_{j=1}^{q} \left(\sum_{i=1}^{r_j} (p_{ji} - \hat{p}_{ji})^2 \right), \quad k = \sum_{j=1}^{q} r_j, \tag{2.1}$$

where q is the number of creep curves; r_j is the number of readings for the creep curve with the number j; $p_{ji} = p_j(t_i)$, $\hat{p}_{ji} = \hat{p}_j(t_i)$ are the theoretical and experimental values of creep strain at the moment of time t_i.

Let us examine system (1.6) more closely. We obtain estimates of the characteristics of steady-state flow C and n by the least squares method, in accordance with [2, 17]. We sill use the method of nonparametric smoothing [18] to determine the material constants F, m, and β. This method makes it possible to smooth empirical observations and their derivatives when certain restrictions are placed on the monotonicity and smoothness of the given relations, without resort to the methods of parametric identification. We use the first equation of system (1.6) to obtain the relation $\omega = 1 - \sigma(C/\dot{p})^{1/n}$, which we then use to find the values $\omega_{ji} = \omega_j(t_j)$ after employing nonparametric smoothing to obtain $\dot{p}_{ji} = \dot{p}_j(t_j)$. Using the latter method to also find ω_{ji}, we obtain estimates of the derivatives $\dot{\omega}_{ji}$. By taking the logarithm, we reduce the equation for the structural parameter ω to a linear regression equation. We then find estimates of F, m, and β by the least squares method on the basis of the available values of $\dot{\omega}_{ji}$, $\dot{p}_{ji}$ (an approximation of the first order with respect to smoothness or closeness [19]). A somewhat better approximation is obtained by the given method when only the values found for β are used and estimates $\tilde{F}_j = F\sigma_j^m$ for each σ_j are determined differently. Integrating system (1.6) with constant σ_j for an arbitrary moment of time t_i, we obtain

$$\tilde{F}_j (C\sigma_j^n)^\beta t_i = \int_0^{\omega_i} \frac{(1-\omega)^{n\beta} d\omega}{[1-(1-\omega)^n]^\beta}. \tag{2.2}$$

Now, determination of $\tilde{F}_j$ by the least squares method requires calculation of the integral in (2.2) from 0 to ω_i for each t_i. The results converge at $\beta < 1$ and a singularity is obtained at $\omega = 0$. We will use the method of isolating singularities in an improper integral [15] to calculate the above integral. We estimate F and m from values of $\tilde{F}_j$ corresponding to certain σ_j.

We obtain estimates of the strength parameters W_* and φ by means of the relation

$$W_{*j} = \int_0^{t_{*j}} \left(\frac{\sigma_j}{1-\omega} \right)^\varphi \dot{p} dt, \tag{2.3}$$

where the integral is not expressed in finite form. Here, the problem consists of determining the value of φ at which the values of the integrals W_{*j} are close to one another for several levels of σ_j. As a measure of closeness, we take the coefficient of variation [17]

$$V(W_*) = \sqrt{D(W)}/|W_*|,$$

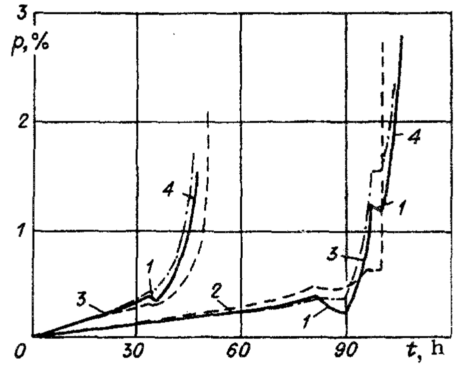

Fig. 1

where $W_* = (\sum_{j=1}^{q} W_{*j})/q$ is the mathematical expectation; D(W) is the dispersion, while we find φ from the condition of the minimum of the functional

$$V\left(W_*(\varphi)\right) = \sqrt{D\left(W(\varphi)\right)} / \left| W_*(\varphi) \right| \to \min.$$

We will solve the above optimization problem by the gradient descent method [20]. We choose φ_0 as the initial approximation, finding this quantity by analyzing empirical data with the use of an equation $W_* = \sigma^{\varphi_0} p_*$ obtained from (2.3) with $\omega(t) = 0$. We will calculate the integral in (2.3) numerically.

Taking the approach described above appreciably simplifies identification of the material constants of model (1.7). In contrast to system (1.6), in the present case we obtain $\tilde{F}_j\,(C\sigma_j^n)^\beta t_i = [1 - (1 - \omega_i)^{n\beta+1}]/(n\beta + 1)$ instead of (2.2) and need not resort to numerical integration.

There is also no need to use numerical integration to determine th strength parameters W_* and φ, since system (1.7) can be integrated analytically. Thus, instead of (2.3) we obtain

$$W_{*j} = \frac{\sigma_j^{\varphi+n-m-n\beta}}{FC^{\beta-1}(n\beta - n + 1 - \varphi)} \{1 - [1 - (n\beta + 1)\,F\sigma_j^{m+n\beta} C^{\beta} t_{*j}]^{(n\beta-n+1-\varphi)/(n\varphi+1)}\}.$$

3. Let us compare models (1.6) and (1.7). We do this by calculating measures of adequacy of approximation (2.1) on the basis of empirical data on the tension of specimens under creep conditions. We used two materials; alloy D16T2 (Fig. 3, tests performed for $\sigma = 320$, 330 MPa (lines 1 and 2), T = 170°C [7]) and steel 12Kh18N10T (Fig. 4, $\sigma = 40$, 50, 60, and 80 MPa (lines 1-4), T = 850°C [2]). Measures of adequacy (2.1) of models (1.6) and (1.7) turned out to be equal to $5.18 \cdot 10^{-6}$ and $3.72 \cdot 10^{-5}$ for alloy D16T2 and $1.24 \cdot 10^{-5}$ and $1.59 \cdot 10^{-5}$ for steel 12Kh18N10T. Use of the Fisher criterion [17] for a significance level $\alpha = 0.05$ showed that the difference in these measures is significant for alloy D16T2 but insignificant for steel 12Kh18N10T. Thus, it is best to use system (1.6) for materials in which the stages of steady -state flow and softening are of roughly the same duration. The results of the approximation are shown in Figs. 3 and 4, where the experimental data is represented by solid lines, the calculation performed with (1.7) is represented by dashed lines, and the calculation performed with (1.6) is represented by dot-dash lines. Estimates of the material constants of steel 12Kh18N10T were obtained using experimental creep curves corresponding to $\sigma = 40$, 50, and 60 MPa, while the theoretical curve – corresponding to $\sigma = 80$ MPa – is the result of prediction with model (1.7). For steel 12Kh18N10T, the boundary between the brittle and ductile fracture regions turned out to be equal to $\sigma_k = 141.4$ MPa.

The validity of model (1.6) for the case of a variable stress was checked on the basis of results of creep tests of steel ÉI698 at T = 750°C [21]. Estimates of the material constants were obtained using experimental creep curves corresponding to stresses that remained constant to failure. The results of the calculations are shown in Fig. 5, where the experimental data is represented by solid lines, the results calculated with model proposed in [21] are represented by dashed lines, and the results calculated with model (1.6) are represented by dot-dash lines. Lines 1-4 correspond to 0, 380, 430, and 480 MPa.

We calculated the scatter (dispersion) [17] of the experimental values of time to rupture and creep strain at failure relative to the corresponding theoretical values found on the basis of system (1.6) and the systems of equations proposed in [1, 2, 4]. Use of the Fisher criterion for the significance level $\alpha = 0.05$ showed that the difference between the respective degrees of scatter is insignificant, i.e. the proposed model describes experimental data on rupture strength at least as accurately as well-known models. The advantage of (1.6) lies in the more accurate description of the strain properties at the softening stage, since the material constants F, m, and β are determined from the entire creep curve. In the studies cited above, the material constants of the models were calculated from the empirical relation $\dot{p}_{min}(\sigma)$, $t_*(\sigma)$, and $p_*(\sigma)$.

Thus, the proposed mathematical model makes it possible to evaluate the stage of steady-state flow in creep and describe the case of deformation in which the creep curve has a vertical or inclined asymptote at the moment of fracture. It was shown that the model leads to a linear damage summation law with weight factors dependent on the loading history. The model makes it possible to describe the nonmonotonic character (with one or two local extrema) of the stress dependence of creep strain at the moment of fracture.

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